

Serie n°1:

Exo1: On tire au hasard deux boules, blanche et noire, dans deux boîtes A et B. On note X la variable aléatoire "nombre de boules dans la boîte A", et Y la variable aléatoire "nombre de boîtes vides".

- 1/ Trouver la loi de la v.a X
- 2/ " " " " Y
- 3/ Trouver la loi conjointe du vecteur (X, Y) , En deduire les lois marginales
- 4/ Déterminer la fonction de Répartition Conjoints
- 5/ Déduire les fonctions de répartition marginales

Exo2: Une urne contient 4 boules blanches, 2 boules noires et 4 boules rouges, on extrait au hasard 3 boules de cette urne sans remise.

On note X la v.a. "le nombre de boules blanches" et Y la v.a. "le nombre de boules noires" figurant parmi les trois boules tirées

- 1/ Trouver la loi conjointe de (X, Y)
- 2/ Déterminer les lois marginales de X et de Y .
- 3/ Calculer la probabilité $P(1 \leq X \leq 2, Y \geq 2)$.

et $P(X \leq 1, 0 \leq Y \leq 1),$

et $P(1 \leq X \leq 2, 0 \leq Y \leq 1)$

Concexion Serie $N=1$

Exo 1:

$$\mathcal{L} = \left\{ \left(\begin{smallmatrix} A \\ B \end{smallmatrix}, \begin{smallmatrix} B \\ N \end{smallmatrix} \right), \left(\begin{smallmatrix} A \\ N \end{smallmatrix}, \begin{smallmatrix} B \\ B \end{smallmatrix} \right), \left(\begin{smallmatrix} A \\ NB \end{smallmatrix}, \begin{smallmatrix} B \\ \end{smallmatrix} \right), \left(\begin{smallmatrix} A \\ \end{smallmatrix}, \begin{smallmatrix} B \\ NB \end{smallmatrix} \right) \right\}$$

① ② ③ ④

1/ $X(\omega) = \{0, 1, 2\}$

x_i	0	1	2	Σ
p_i	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$	1
	$P(④)$	$P(② \cup ③)$	$P(③)$	

2/ $Y(\omega) = \{0, 1\}$

y_i	0	1	2
p_i	$\frac{1}{2}$	$\frac{1}{2}$	1
	$P(③ \cup ④)$	$P(② \cup ③)$	

3/ $X(\omega) \times Y(\omega) = \{(0,1), (1,0), (2,1)\}$

$$P_{X,Y}(0,1) = P(X=0, Y=1) = P(④) = \frac{1}{4}$$

$$P_{X,Y}(1,0) = P(X=1, Y=0) = P(① \cup ②) = \frac{1}{2}$$

$$P_{X,Y}(2,1) = P(X=2, Y=1) = P(③) = \frac{1}{4}$$

$$* P(X=0) = \sum_{j=0}^1 p_{0j} = p_{00} + p_{01} = \frac{1}{4}$$

et même pour les autres cas et de la loi de Y aussi

$X \backslash Y$	0	1	$p_{i\cdot}$
0	0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{2}$	0	$\frac{1}{2}$
2	0	$\frac{1}{4}$	$\frac{1}{4}$
$p_{\cdot j}$	$\frac{1}{2}$	$\frac{1}{2}$	1

4) ① si $x < 0$ $F_{X,Y}(x,y) = P(\emptyset) = 0$

② si $0 \leq x < 1$, $0 \leq y < 1$, $F_{X,Y}(x,y) = P(\emptyset) = 0$

③ si $0 \leq x < 1$, $y \geq 1$, $F_{X,Y}(x,y) = p_{01} = \frac{1}{4}$

④ si $0 \leq x < 1$, $0 \leq y < 1$, $F_{X,Y}(x,y) = p_{1,0} = \frac{1}{2}$

⑤ si $1 \leq x < 2$, $0 \leq y < 1$, $F_{X,Y}(x,y) = p_{01} + p_{1,0} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

⑥ si $x \geq 2$, $y \geq 0$, $F_{X,Y}(x,y) = p_{01} + p_{1,0} + p_{2,1} = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

5/ si $y < 0$ $F_Y(y) = \lim_{x \rightarrow +\infty} F_{X,Y}(x,y) = F_{X,Y}(+\infty, y) = P(\emptyset) = 0$ (cas ①)

si $0 \leq y < 1$ $F_Y(y) = F_{X,Y}(+\infty, y) = F_{X,Y}(+\infty, y) = p_{1,0} = \frac{1}{2}$

si $y \geq 1$ $F_Y(y) = F_{X,Y}(+\infty, y) = p_{01} + p_{1,0} + p_{2,1} = 1$

fonction de répartition de X

$$\text{si } x < 0 \quad F_X(x) = F_{X,Y}(x, +\infty) = P(\emptyset) = 0 \quad (\text{cas 1})$$

$$\text{si } 0 \leq x < 1 \quad F_X(x) = P_{0,1} = \frac{1}{4} \quad (\text{cas 2})$$

$$\text{si } 1 \leq x < 2 \quad F_X(x) = \frac{3}{4} \quad (\text{cas 3})$$

$$\text{si } x \geq 2 \quad F_X(x) = 1 \quad (\text{cas 4})$$

Exo 2, Card(Ω) = $C_{10}^3 = 120$. $\Omega = \{(B, B, B), (B, B, N), (B, N, B), \dots\}$

$$X(x) \times Y(y) = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (3,0)\}$$

$$P_{00} = P(X=0, Y=0) = P((X,Y) \rightarrow (0,0)) = P((R,R,R)) = \frac{C_4^3}{C_{10}^3} = \frac{4}{120} = \frac{1}{30}$$

$$P_{01} = P(X=0, Y=1) = P(\{(N,R,B), (R,N,B), (R,R,N)\})$$

$$= \frac{C_2^1 \times C_4^2}{C_{10}^3} = \frac{4 \times 6}{120} = \frac{6}{30}$$

$x \backslash y$	0	1	2	h_{i0}
0	$1/30$	$3/30$	$1/30$	$5/30$
1	$6/30$	$2/30$	$1/30$	$15/30$
2	$6/30$	$3/30$	0	$9/30$
3	$0/30$	$0/30$	0	$1/30$
$P_{\cdot j}$	$14/30$	$10/30$	$2/30$	1

$$3/ \quad P(1 \leq X \leq 2, 0 \leq Y \leq 1)$$

$$= P(X=1, Y=0) + P(X=1, Y=1) + P(X=2, Y=0) + P(X=2, Y=1)$$

$$= P_{10} + P_{11} + P_{20} + P_{21}$$

$$= 23/30$$

$$P(X \leq 1, 0 \leq Y \leq 1)$$

$$= P_{00} + P_{01} + P_{10} + P_{11} = 18/30$$

$$P(1 \leq X \leq 2, Y \geq 2)$$

$$= P(1 \leq X \leq 2, Y=2)$$

$$= P_{12} + P_{22} = 1/30 + 0 = 1/30$$

fonction de répartition conjointe =

- Si $x < 0$ ou $y < 0 = F_{x,y}(x,y) = P(\emptyset) = 0$ --- (1)
- Si $0 \leq x < 1$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} = \frac{1}{30}$
- Si $1 \leq x < 2$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} + P_{10} = \frac{7}{30}$
- Si $2 \leq x < 3$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} + P_{10} + P_{20} = \frac{13}{30}$
- Si $x \geq 3$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} + P_{10} + P_{20} + P_{30} = \frac{14}{30}$ --- (6)
- Si $0 \leq x < 1$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} = \frac{4}{30}$
- Si $1 \leq x < 2$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{10} + P_{11} = \frac{18}{30}$
- Si $2 \leq x < 3$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{10} + P_{11} + P_{20} + P_{21} = \frac{27}{30}$
- Si $x \geq 3$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{10} + P_{11} + P_{20} + P_{21} + P_{30} + P_{31} = \frac{28}{30}$ --- (7)
- Si $0 \leq x < 1$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} = \frac{5}{30}$ --- (2)
- Si $1 \leq x < 2$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} = \frac{20}{30}$ --- (3)
- Si $2 \leq x < 3$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} + P_{20} + P_{21} + P_{22} = \frac{29}{30}$ --- (4)
- Si $x \geq 3$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} + P_{20} + P_{21} + P_{22} + P_{30} + P_{31} + P_{32} = 1$ --- (5)
- Si $0 \leq x < 1$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} = \frac{1}{30}$
- Si $0 \leq x < 1$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} = \frac{4}{30}$
- Si $0 \leq x < 1$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} = \frac{5}{30}$ --- (2)
- Si $1 \leq x < 2$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} + P_{10} = \frac{7}{30}$
- Si $1 \leq x < 2$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{10} + P_{11} = \frac{18}{30}$
- Si $1 \leq x < 2$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} = \frac{20}{30}$ --- (3)
- Si $2 \leq x < 3$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} + P_{10} + P_{20} = \frac{13}{30}$
- Si $2 \leq x < 3$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{10} + P_{11} + P_{20} + P_{21} = \frac{27}{30}$
- Si $2 \leq x < 3$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} + P_{20} + P_{21} + P_{22} = \frac{29}{30}$ --- (4)
- Si $x \geq 3$, $0 \leq y < 1 = F_{x,y}(x,y) = P_{00} + P_{10} + P_{20} + P_{30} = \frac{14}{30}$ --- (6)
- Si $x \geq 3$, $1 \leq y < 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{10} + P_{11} + P_{20} + P_{21} + P_{30} + P_{31} = \frac{28}{30}$ --- (7)
- Si $x \geq 3$, $y \geq 2 = F_{x,y}(x,y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} + P_{20} + P_{21} + P_{22} + P_{30} + P_{31} + P_{32} = 1$ --- (5)

fonction de répartition marginales =

$$X(\omega) = \{0, 1, 2, 3\}; F_X(x) = \lim_{y \rightarrow +\infty} F_{x,y}(x,y)$$

Si $x < 0 = F_X(x) = \lim_{y \rightarrow +\infty} F_{x,y}(x,y) = F(n, +\infty) = P(\emptyset) = 0$ (le cas 1)

Si $0 \leq x < 1 = \frac{5}{30} \rightarrow$ (le cas 2)

Si $1 \leq x < 2 = \frac{20}{30} \rightarrow$ (le cas 3)

Si $2 \leq x < 3 = \frac{29}{30} \rightarrow$ (le cas 4)

Si $x \geq 3 = 1 \rightarrow$ (le cas 5)

$$Y(\omega) = \{0, 1, 2\}; F_Y(y) = \lim_{x \rightarrow +\infty} F_{x,y}(x,y)$$

Si $y < 0 = P(\emptyset) = 0$ (le cas 1)

$$\begin{aligned} \text{Si } 0 \leq y < 1 &= \frac{14}{30} \rightarrow (\text{le cas } 6) \\ \text{si } 1 \leq y < 2 &= \frac{28}{30} \rightarrow (\text{le cas } 7) \\ \text{Si } y \geq 0 &= 1 \rightarrow (\text{le cas } 5) \end{aligned}$$

Fonctions de répartition conjointe:

- 1) Si $x < 0$ ou $y < 0$: $F_{xy}(x, y) = P(\emptyset) = 0$.
- 2) Si $0 \leq x < 1, 0 \leq y < 1$: $F(x, y) = P_{00} = \frac{1}{30}$.
- 3) Si $1 \leq x < 2, 0 \leq y < 1$: $F(x, y) = P_{10} + P_{00} = \frac{7}{30}$.
- 4) Si $2 \leq x < 3, 0 \leq y < 1$: $F(x, y) = P_{00} + P_{10} + P_{20} = \frac{13}{30}$.
- 5) Si $x \geq 3, 0 \leq y < 1$: $F(x, y) = \frac{24}{30}$.
- 6) Si $0 \leq x < 1, 1 \leq y < 2$: $F(x, y) = P_{01} + P_{00} =$
- 7) Si $1 \leq x < 2, 1 \leq y < 2$: $F(x, y) = P_{00} + P_{01} + P_{10} + P_{11} = \frac{18}{30}$.
- 8) Si $2 \leq x < 3, 1 \leq y < 2$: $F(x, y) = P_{00} + P_{01} + P_{10} + P_{11} + P_{20} + P_{21} = \frac{27}{30}$.
- 9) Si $x \geq 3, 1 \leq y < 2$: $F(x, y) = P_{00} + P_{01} + P_{10} + P_{11} + P_{20} + P_{21} + P_{30} + P_{31} = \frac{28}{30}$.
- 10) Si $0 \leq x < 1, 2 \leq y$: $F(x, y) = P_{00} + P_{01} + P_{02} = \frac{5}{30}$.
- 11) Si $1 \leq x < 2, 2 \leq y$: $F(x, y) = P_{00} + P_{01} + P_{02} + P_{10} + P_{11} + P_{12} = \frac{20}{30}$.
- 12) Si $2 \leq x < 3, 2 \leq y$: $F(x, y) = \frac{29}{30}$.
- 13) Si $x \geq 3, y \geq 2$: $F(x, y) = 1$.

14) Si